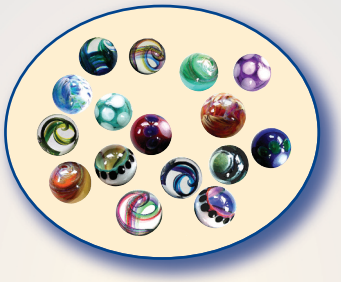
Math4Minors Presents:

EDUCATORS GUIDE:

Miss Penny Says *Prove It!*



A comprehensive educational resource aimed at demystifying Common Core Standards and promoting in-depth understanding of key math concepts for educators, parents and kids.

Lynda Brennan

Math MileMarkers®

Calm, Command and Conquer the Curriculum

Math MileMarkers Educational Guide/Children's Book Series

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Miss Penny Says *Prove It!*

This Math MileMarkers original story about Miss Penny and her lively class outlines so many important learning opportunities that are essential for a child to develop a true numerical understanding. Math4Minors sets out to streamline the most recent information available, helping parents and teachers get to the core of what children need to know and understand in order to be successful in today's world. Our goal is to put math inside a child's real world through relatable stories, and then uncover the stages of learning that typically evolve as their learning progresses.

The story of Miss Penny highlights several foundational skills that children will confront on their mathematical journey, including counting and cardinality, the developmental progression for number knowledge, estimation, as well as a brief encounter with the concept of base ten. It's important to understand that there are no shortcuts on this mathematical journey. Children must experience each Math MileMarker, each critical phase, and each developmental progression as they move toward their own in-depth understanding of what numbers really are and what they represent. Children can only develop this essential foundational knowledge of numbers if they spend time exploring numbers, counting objects, putting numbers together, and taking numbers apart. This can be time intensive, but will certainly be time well spent.

Let's take a look at what some of the most current research is suggesting. The US Department of Education, in its November 2013 report *What Works Clearinghouse: Teaching Math to Young Children*, noted that children display strong interests in math long before they enter school. It is our job to foster this natural interest by providing multiple experiences for them to explore numbers. The research presented provides evidence that a child's early math achievement can often predict future reading achievement. It is believed that foundational skills in number and operation may prepare children to approach reading skills with greater success. With this in mind, the panel strongly suggested that more emphasis be placed on math instruction and made the following five recommendations:[1]

Recommendations

- Teach number and operations using a developmental progression.
- Teach geometry, patterns, measurement, and data analysis using a developmental progression.
- Use progress monitoring to ensure that math instruction builds on what each child knows.
- Teach children to view and describe their world mathematically.
- Dedicate time each day to teaching math, and integrate math instruction throughout the school day.

These recommendations will be further explored in the educational components that follow. It is clear that time spent understanding numbers and the relationship between numbers will give children a toolbox of skills that will serve them for life. State

and Common Core Learning Standards call for students to apply math to real-world situations and not just implement a series of procedures. Research proves that there are developmental progressions that must be adhered to in order to build the foundational skills that young learners need to become critical thinkers and to have a solid understanding of the number system.

Closing Thoughts:

Recently published teacher resources such as *Cowboys Count, Monkeys Measure* and *Princesses Problem Solve and Number Stories: Using Children's Literature to Teach Young Children Number Sense*, highlight the value of incorporating storytelling into math instruction as a means of engaging students in mathematical thinking. Teachers who work with children will confirm that creating relatable images and stories helps foster rich math inquiry and can launch meaningful discussions in the classroom. Math MileMarkers make the link to content easy for teachers and parents and enjoyable for kids.

Our goal must be to create great problem solvers; to emphasize the power and satisfaction of thinking through rich, thought-provoking questions; and to offer appropriate challenges that move children from their current milestone to the next stop along the way. Instituting daily opportunities to talk about math using common math vocabulary is essential to this development. Only in doing this will we prepare children for a successful journey and encourage the love of learning that will eventually help them solve and make sense of the complex problems they will face further down the road.

The pages that follow provide an educational guide that will help parents and teachers capitalize on the learning opportunities imbedded within our story *Miss Penny Says **Prove It**!* Recommendations for instruction will be highlighted in the following sections:

- Standards for Mathematical Practice
- Counting and Cardinality & Operations and Algebraic Thinking
- Number and Base Ten
- Math MileMarkers: Structures for Learning

MileMarkers Math Talk and Storyboard Activities are available on our website at www.mathmilemarkers.com.

Standards for Mathematical Practices

The Standards for Mathematical Practice outline specific expertise that students at all levels should seek to obtain in order to become proficient at math and successful lifelong learners. Although often overlooked, these standards need to be intertwined with the content standards, which address specific skills that children should know and be able to do at each grade level. The Mathematical Practice Standards are key elements in the learning progression as they promote true understanding versus just a series of procedures. As stated earlier, our goal must be to encourage children to become good thinkers and problem solvers. To do that, we must combine content standards with practice standards in a seamless way.

The Standards for Mathematical Practice (MP) combine the National Council of Teachers of Mathematics (NCTM) process standards with the proficiency standards outlined by the National Research Council in their report *Adding It Up.* The eight MP standards are outlined below:[2]

#	MATHEMATICAL PRACTICE STANDARDS
MP.1	Make sense of problems and persevere in solving them.
MP.2	Reason abstractly and quantitatively.
MP.3	Construct viable arguments and critique the reasoning of others.
MP.4	Model with mathematics.
MP.5	Use appropriate tools strategically.
MP.6	Attend to precision.
MP.7	Look for and make use of structure.
MP.8	Look for and express regularity in repeated reasoning.

Why is this important?

Miss Penny Says Prove It! provides educators with a comfortable pathway toward learning to use the Standards for Mathematical Practice in their everyday classroom. This simple story about Bobby's marble collection clearly unleashed an opportunity for the class to make sense of a problem and persevere in solving it. Mathematically proficient students start with what they know and connect ideas that are given, in an effort to find an entry point into a question. These students habitually ask themselves, does this make sense?

Early in the story, Miss Penny asks her students to guess how many marbles are in Bobby's collection. As a young teacher in a kindergarten classroom many years ago, I can recall the morning math activity in which students were asked to estimate how

many objects were in a jar. The answers I got back were generally absurd. I can remember thinking, well, this is a total waste of time. What I came to understand as that first year progressed, was that the more time children spent counting objects, making connections with the numbers they said and the quantities they represent, the closer the estimates became. You see, in this early stage of learning, it's less about estimation and more about helping children to think about quantity and the number names we give to each quantity.

Of course the idea of estimation evolves through the elementary years and becomes a key strategy that all good learners use to assess the reasonableness of their answers, a skill that they will use for life. In the preschool and primary grades, estimating numbers gives teachers insight into a child's true understanding of numbers. In other words, does the child have a sense of what that number is and what it represents, or is it simply a guess? Can a child subitize small groups of objects and use that information to inform his or her response to a greater question?

In the story, when Rosa guesses "two" as she looks at this pile of marbles for the first time, she clearly demonstrates no conservation of numbers. Teachers need to be in tune with where each student is on this spectrum of learning. As stated earlier, the only solution to this problem is time; time to explore, to count, to recount, to reorganize and count again, to break apart and put back together a number in multiple ways. Time dedicated to multiple experiences, beginning with small groups of objects and working up to larger groups of objects, is the only way for children to truly gain an in-depth understanding of numbers.

Next, as the marbles moved from Miss Penny's hands to the bucket and eventually to the lunch bag, students were exposed to quantitative reasoning. As you can see, the story was not just about how to compute numbers, but instead, as called for in MP.2, it offered a chance for children to create a coherent representation of the number sixteen and gave real meaning to the units involved in making up that number.

As the story progressed, the words *"Prove it"* took on a significant meaning. What Miss Penny was really asking students to do was to rationalize and justify their answers. Miss Penny, during the course of her lesson, promoted "math talk," simple conversations that allowed her students to find multiple pathways to the solution and to engage themselves in related vocabulary. By doing such, she incorporated Practices MP.3 and MP.6.

It is important that teachers don't overlook the significance of the Standards for Mathematical Practice. These, combined with the content standards, provide the broad reach that students need to build strong foundational knowledge.

Counting and Cardinality & Operations and Algebraic Thinking

Very often adults assume that if a child is able to rote count, then the child *knows* his or her numbers. After all, many children by age three can recite their numbers like champions, but do they really know what numbers are or what they represent?

As suggested by the National Research Council of the National Academies, mathematics offers a powerful opportunity to understand the world. The foundational skills that children come across in preschool and the early primary grades, in most cases, are never addressed in the curriculum by the time a child gets to grade 3. Without owning numerical concepts or having a solid qualitative understanding of each number, a student will never be able to combine and break apart numbers and thus will have more difficulty making sense of later math. These missing building blocks of knowledge create a wall of obstacles that can hinder a child's future growth in math. As parents and educators, it is essential that we provide adequate opportunities for children to learn and understand numbers by giving them multiple chances in and out of school to practice counting and exploring numbers.

At a recent parent math workshop I asked parents to answer the question E + J = ?. Initially I had a collection of confused faces staring back at me. After all, E and J had no real meaning to them. How could they combine them if they had no known quantitative value? I had essentially placed the parents in the mindset of a young learner, simulating for them what it is like before one truly knows what a number is and what it represents.

Eventually, parents realized that there could be value in these numbers if they started at the beginning and simply counted up. With fingers as tools, parents came to understand a,b,c,d,e as 1,2,3,4,5, thus E equals 5, or has a value of 5. Next they went to J. Some parents started at E or 5 and counted up, while others started at the beginning, having far more confidence in their one-to-one correspondence approach. When challenged with the value of the letter P, they again counted up or counted on, depending on where their level of confidence and understanding was at that moment. You could almost see the thinking in progress as parents scrunched eyebrows, made notes on papers, and discussed strategies among themselves to solve these simple problems. It wasn't until they encountered J on several occasions that they finally came to realize that J equals 10 and could use it without recounting. At this point a calm came across most faces in the room, as they had learned to solve simple equations using the make-believe number J. Parents now owned this knowledge. They could visualize J and the ten objects that J represented, and from that point forward most could successfully apply it.

Many parents, without even realizing it, already support this quest for knowledge of numbers. Take, for example, the child who holds up three fingers when asked how old he or she is. Although initially children fumble with getting three fingers in the air, eventually they can hold onto the value of three.

As you can see, knowing a number's name and understanding its true quantitative value are two very different things. Until a child owns that knowledge and can "prove"

a number's worth, or shall we say "value," all mathematical processes that follow will have no true meaning. Taking that one step further, children must have opportunities to see beyond just three fingers as the image of three. To display true understanding of the number three, children must be able to count three blocks or paperclips out of a larger pile. In the early stages of number development many children will count right past a number. With practice, using multiple items like blocks, buttons, pencils, or dots on cards, children eventually recognize and are able to conceptualize a group of objects that make up each number. They learn to visualize each quantity without the help of concrete objects.

As the progression of knowledge continues, children learn to subitize or see numbers in small groups that they can immediately recognize as a specific total. They can look at four dots on a card and say "four" without counting. Regardless of the math resource you choose to embrace, one common theme remains: a child's understanding of numbers evolves across a series of stages or learning progressions, one building on the next.

The US Department of Education established a chart that outlines what they refer to as the Developmental Progression for Number Knowledge. This chart can be quite effective at identifying where a child is within the stated learning progression. With this knowledge, educators can create developmentally appropriate instruction for each child, moving back to prerequisite skills when needed and moving forward to the appropriate next step only when the child achieves mastery of the skill at hand. The Developmental Progression document is a road map for teachers and encourages the mastering of each skill with small numbers before moving on to larger numbers.[3]

In our story, the children in Miss Penny's room were clearly not all on the same page in terms of their numerical knowledge. The math conversations that transpired provided Miss Penny with insight into where each child might be with regard to the learning progressions.

Many notable primary math authors and programs promote similar phases of learning that are seen as essential for building number sense and mastery of the number system. All research points to the fact that learning numbers requires a series of developmental steps and multiple opportunities for practice. This only confirms the significance of each Math MileMarker encountered by children on their mathematical journey.

It is important to take note of the fact that the math strand entitled Counting and Cardinality ceases to exist by grade 1. The cardinality skills in Pre-K and kindergarten CCLS progress into standards related to Operations and Algebraic Thinking and Number and Base Ten. The CCLS standards found in grades 1, 2, and beyond assume that children know and own the value of numbers and that they are prepared to work with numbers in new ways.

Clearly the new CCLS and state standards have raised the bar for children, creating higher expectations and promoting critical thinking skills. Interestingly enough, most kindergarteners that I have worked with have responded well to this increased challenge. Most children have been leaving kindergarten able to rote count to a hundred. And, as the CCLS standards outline, most children have gained a solid understanding of the number five and can build and take apart numbers before they enter first grade. With expectations clearly defined, more children are entering grade 1 ready to expand on their knowledge of numbers, ready to take on the next step. For those not developmentally there just yet, and it's important to know that not all will be, the Developmental Progression for Number Knowledge provides a road map for parents and educators. Like Miss Penny, we need to be in tune with where children stand on their personal progression or journey. As parents and educators, we need to understand the numerical progressions and encourage MileMarker pit stops that allow children to master each skill and concept before moving on. In doing so, we will create a smooth road ahead for all levels of learners.

US DEPARTMENT OF EDUCATION: SPECIFIC DEVELOPMENTAL PROGRESSION FOR NUMBER KNOWLEDGE

Subitizing Small-Number Recognition	Subitizing refers to a child's ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of quantity (e.g., two eyes, two hands, two socks) labeled with the same number word, as well as nonexamples labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three. A child is ready for the next step when, for example, he or she is able to see one, two, or three stickers and immediately—without counting—state the correct number of stickers.
Meaningful Object Counting	Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is the cardinality principle). A child is ready for the next step when, for example, if given five blocks and asked "How many?" he or she counts by pointing and assigning one number to each block: "One, two, three, four, five," and recognizes that the total is "five."
Counting-Based Comparisons of Collections Larger than Three	Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of the two collections (e.g., "seven" items is more than "six" items because you have to count further). A child is ready for the next step when he or she is shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., "nine" bears is more).
Number-After Knowledge	Familiarity with the counting sequence enables a child to have number-after knowledge—i.e., to enter the sequence at any point and specify the next number instead of always counting from one. A child is ready for the next step when he or she can answer questions such as, "What comes after five?" by stating "five, six" or simply "six" instead of counting "one, two,…six."
Mental Comparisons of Close or Neighboring Numbers	Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers. A child has this knowledge when he or she can answer questions such as, "Which is more, seven or eight?" and can make comparisons of other close numbers.
Number-After Equals One More	Once children can mentally compare numbers and see that "two" is one more than "one" and that "three" is one more than "two," they can conclude that any number in the counting sequence is exactly one more than the previous number. A child is ready for the next step when he or she recognizes, for example, that "eight" is one more than "seven."

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Number and Base Ten

As children grow in their understanding of numbers, the idea of place value can naturally unfold given the right set of experiences and some well-directed math talk. From the very start, the focus must be on building to ten. Simple structures like teacher-made ten-frames; manipulatives like base ten blocks, toy cubes, and other counting objects; as well as daily practice via various desktop and online games that focus on building ten can help children gain an appreciation for the magnitude of the number ten and the place of nobility it holds within our number system.

Keep in mind that in kindergarten, children are asked to practice composing and decomposing five objects in preparation for this big idea of "ten." Once the idea of ten is established and children understand that all numbers can be put together and taken apart, the foundational link for numeracy is in place. All numbers beyond that point fall into the pattern of ever-evolving numbers that are related to each other. Children begin to see eleven as one ten and one more; thirty-six becomes three tens and six more, and 1003 becomes one hundred tens and three more, and so it grows.

Miss Penny's class was composed of a broad range of learners, each approaching the number sixteen based on his or her own level of comfort. Bobby clearly displayed his understanding of one-to-one correspondence. Maggie was the first to establish a group of ten marbles before continuing on to sixteen. Regina took that idea one step further when she saw the ten marbles as one group of ten and six more and then proceeded to add these two quantities to get to sixteen.

It is by no small chance that the number sixteen was chosen for this story. The final illustration in the book consists of pictures that were drawn by students that I have worked with. When given the task to draw sixteen, each of them had a multitude of ideas about the number sixteen and how they could represent that quantity. Most of them simply did not want to settle for just one drawing on the page. I was happy to see them see the number sixteen come to life in their minds as sixteen ones; one ten and six more; three groups of five and one more; a number line with many different ways to jump from zero to their final destination of sixteen; sixteen objects, sticks, bars, and units all carefully configured to make sixteen. The students who completed this body of work ranged in age from two to ten. Little Matthew and Christopher, at two years old, together combined three handprints and a finger to build sixteen. Although not completely cognizant of what the value of sixteen really meant at merely two years of age, these young mathies were already showing interest in numbers, reciting numbers aloud as they joined in the fun. We need to capitalize on these encounters and connect them to the real math proficiencies that they will evolve into. In other words, don't miss the teachable moments that have sometimes lost their place in classrooms bogged down by new standards. It is in doing this real-life teaching for everyday kids that we can help children know and utilize their own thinking, the most powerful math tool there is, and that early math students can learn to truly understand the power of ten.

Math MileMarkers: Structures for Learning

The section that follows provides teachers and parents with an easy-to-read outline that highlights the most important aspects of the learning standards addressed within the story. The Math MileMarkers charts break things down into three components: the actual *Learning Standards*, which present the Common Core Learning Standard addressed and the related standard and grade level; *True Meanings*, which offers readers an inside look into what the standard is really about, what students should know and be able to do, and various structures that can be used to help them find meaning; and *Math MileMarkers*, which offers sample questions and age-appropriate tasks related to each of the standards addressed. In other words, how might the students encounter this content in a class or testing situation?

The Common Core Learning standards have raised the bar in terms of what students are expected to be able to know and do at each grade level. The good news is that in doing so, the writers of Common Core have provided much needed clarity and greater definition of the curriculum for teachers and students. And, they have established a clear, progressive path that allows learners to move backwards to specific prerequisite skills when needed, and forward when enrichment opportunities are appropriate.

Please join the Math MileMarkers Professional Learning Community by registering with us at www.mathmilemarkers.com. As a member you will receive periodic updates regarding new products, resources and services that can help *Calm, Command, and Conquer the Curriculum!®*

Common Core Learning Standard

PK.CC.1 Standard: Count to 20

True Meanings	Math MileMarkers®
Grade Level: Pre-Kindergarten **Math Strand: Counting and Cardinality** **Standard #1** **True Meanings:** This standard suggests that all students should be able to rote count to 20 by age 5. It should be noted that many children find the teen numbers difficult to remember, but with practice and song-like approaches, most students at this age can learn to say the numbers in sequential order from 0 to 20. This skill is best assessed via individual student conferences.	**"The 20 Song"**: sung to the tune "I'm a Little Teapot." 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,12, 13, 14, 15, 16, 17, 18, 19, 20 I can count my numbers to 20. **The Count Twenty Math MileMarker Cheer** Note: Hand motions are made in groups of five. Voices remain soft throughout the cheer but become loud, happy voices when cheering the multiples of 5. Students should progress to cheering without the need to repeat after the teacher. This is an excellent transition routine for young children who need to move and be "active" learners—in other words, for all students. Teacher says "one." Students repeat "one." Hands go up to the sky. Teacher says "two." Students repeat "two." Hands go up to the sky. Teacher says "three." Students repeat "three." Hands go up to the sky. Teacher says "four." Students repeat "four." Hands go up to the sky. Teacher yells "five." Students yell "five." Hands go up to the sky. Teacher says "six." Students repeat "six." Hands go out to the side. Teacher says "seven." Students repeat "seven." Hands go out to the side. Teacher says "eight." Students repeat "eight." Hands go out to the side. Teacher says "nine." Students repeat "nine." Hands go out to the side. Teacher yells "ten." Students yell "ten." Hands go out to the side. Teacher says "eleven." Students repeat "eleven." Tilt head to the right. Teacher says "twelve." Students repeat "twelve." Tilt head to the left. Teacher says "thirteen." Students repeat "thirteen." Tilt head to the right. Teacher says "fourteen." Students repeat "fourteen." Tilt head to the left. Teacher yells "fifteen." Students yell "fifteen." Tilt head to the right. Teacher says "sixteen." Students repeat "sixteen." Shrug shoulder up. Teacher says "seventeen." Students repeat "seventeen" Shrug shoulder up. Teacher says "eighteen." Students repeat "eighteen." Shrug shoulder up. Teacher says "nineteen." Students repeat "nineteen." Shrug shoulder up. Teacher yells "twenty." Students yell "twenty." Shrug shoulder up. Teacher says "five." Students say "five." Hands go up to the sky, voices soft. Teacher says "ten." Students say "ten." Hands go out to the side. Teacher says "fifteen." Students say "fifteen." Tilt head to the right. Teacher says "twenty." Students say "twenty." Shrug shoulder up. Everyone rolls their hands and throws their arms up to the sky and yells "Go 20!"

Common Core Learning Standard

PK.CC.3 Standard: Understand the relationship between numbers and quantities to 10; connect counting to cardinality.

a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

c Understand that each successive number name refers to a quantity that is one larger.

True Meanings

Grade Level: Pre-Kindergarten
Math Strand: Counting and Cardinality
Standard #3

True Meanings:

There is no shortcut for this Mile-Marker. Students need opportunities to see numbers, recognize their value, learn the meaning of "equals," and be able to place numbers on the number line in order to gain the in-depth understanding needed to be successful at all future math encounters.

- Numbers grow in magnitude (each successive number represents one more) as they move further down the number line.
- Numbers stay the same. Regardless of the formation the objects are in, they hold and represent the same value.
- The last number you count represents the numerical value of that group.

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Games and Tools to Practice Counting

- Counting concrete objects (cubes, links, buttons, pennies)
- Ready -made games with links or other counters
- Dot cards and Ten Frame Templates
- Picture games (matching numbers with objects)
- Blocks or cube towers or stairs (one more or less)
- Sorting games
- Piano math – numbered fingers
- Reckenrec

Counting and Cardinality Performance Task—PK.CC.3

Title: *Counting Fingers and Cubes*

1 Trace a child's two hands on a large piece of paper.

2 Number each finger from 1 to 10 starting on the left and moving to the right.

3 Ask child to place cubes on each finger as he or she counts out objects.

Note: Emphasis should be on one-to-one correspondence. Students need to learn that each number represents a specific quantity. Three fingers, just like 3 cubes, always have a value of 3.

Common Core Learning Standard

PK.CC.4 Standard: Count to answer "how many?" questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration; given a number from 1–10, count out that many objects.

True Meanings

Grade Level: Pre-Kindergarten
Math Strand: Counting and Cardinality
Standard #4

True Meanings:

This standard builds on standard PK.CC.3. It stresses the concept of identifying "how many" and allows for the introduction of math vocabulary such as equal to and same value, more, less.

It is essential that students be given an opportunity to work with physical objects when exploring the idea of how many. And, facilitators should note that the math conversations that accompany this task are equally as important as the task itself.

Be sure to encourage group conversations with students, modeling how math vocabulary can be used to describe and compare number of objects.

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Counting and Cardinality Task— PK.CC.3. and PK.CC.4

Rings of Marbles

1 Place a small hoop on the table with 9 marbles inside.

2 Ask the child to count marbles by picking up each marble and placing them one by one inside a cup.

3 Ask child ***how many*** marbles are in the cup?

4 Put the same marbles from the cup back into the hoop. Can the child tell you how many marbles are in the hoop now?

5 Did the student recognize that the number did not change?

6 Change quantities, up to 10, displaying marbles in various configurations.

Common Core Learning Standard

Standard K.CC.1—Count to 100 by ones and by tens

True Meanings

Grade Level: Kindergarten
Math Strand: Counting and Cardinality
Standard #1

True Meanings:

In this standard, students are expected to rote count, knowing number names in order, from 0 to 100. Students should be able to count by 10s by saying 0, 10, 20, 30…100. Although this standard does not require the recognition of numerals, it is highly recommended that students be provided with a 100 Chart when practicing this skill, so they can point to numbers as they count. Doing so will promote standards that address concepts like one-to-one correspondence and number recognition.

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Counting and Cardinality Task—K.CC.1

Touch and Talk to 100

Part 1:

Count aloud, pointing to each number on the chart as you say its name. Point and speak as you go.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Part 2:

If you were counting by 10s, what numbers would be missing on the charts below? Count aloud from the start point and say the existing and missing numbers in each line. Point and speak as you go.

Start 10			40	
Start 50				90
Start 60			90	

Common Core Learning Standard

Standard K.CC.4: Understand the relationship between numbers and quantities; connect counting to cardinality.

a Student can say the number names in the standard order, pairing each object with one and only one number name.

b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

c Understand that each successive number name refers to a quantity that is one larger.

True Meanings

Grade Level: Kindergarten
Math Strand: Counting and Cardinality
Standard #4

True Meanings:

The progression within the Counting and Cardinality strand that begins in Pre-K surfaces again in kindergarten with larger numbers. This strand is really about the relationship between sets of objects and the number names that give identity to their value. This is a higher-level skill that requires children to reason about and analyze numbers. Based on CCLS standards, children should be confident in their quantitative understanding of numbers from 0–20 by the end of kindergarten. How do children master this understanding? Practice!

At the very foundation of counting is a child's ability to master one-to-one correspondence, also known as the tagging of numbers. A child's mathematical journey will take him or her naturally to the next developmental milemarker, which addresses the fact that the number of objects does not change when the objects are moved or hidden. This is part of the developmental progression, and like all mathematical concepts, not all children will arrive at the same point of understanding at the same time. How do children master this quantitative understanding? Practice!

The next MileMarker is directly related to standard PK.CC.3. In short, numbers build by exactly one each time, and the numbers nest within each other by this amount. In other words five is made up of four and one more. The numbers four and one nest inside five. This concept is at the core of the future development of part/whole relationships and is fundamental to the understanding of the number system.

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How many stars are shown below? _____

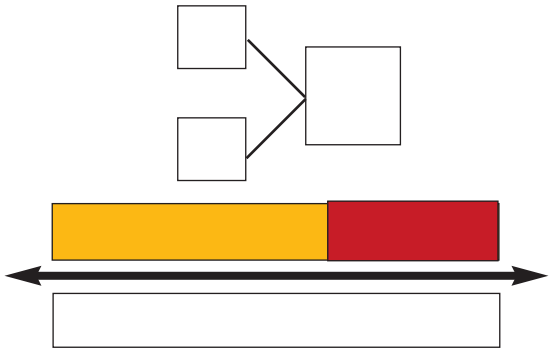
If you had one more star than the stars above, how many stars would you have?

_____ Stars

Concrete Tools to Practice Counting Skills

- Base ten blocks
- Concrete counting objects (cubes, links, buttons, pennies)
- Dot cards and Ten frame templates
- Matching number/pictures games (number and object matching)
- Blocks or cube towers and stairs –up to 20
- Matching, Sorting, Hiding games
- Piano Math
- Number cards
- Models – number bonds, part/whole organizers
- Reckenrec

This is an ideal time to incoporate number bonds and other part/whole templates when solving problems that involve the composition (building) and decomposition (taking apart) of numbers.



Common Core Learning Standard

Standard K.CC.5: Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

True Meanings

Grade Level: Kindergarten
Math Strand: Counting and Cardinality
Standard #5

True Meanings:

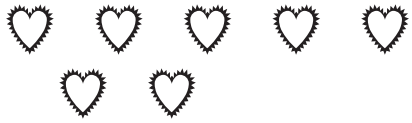
This standard is about a child's ability to keep track of objects when counting them in order to determine how many. It is important to promote counting strategies that support developing learners. Some of these strategies include:

* ***Counting*** a set of objects
* ***Moving*** the objects as they count each
* ***Pointing*** to each object as it is counted
* ***Looking*** without touching when counting
* ***Lining up*** objects in groups of 5 or 10 or rectangular arrays.

Through multiple counting experiences, children must come to the realization that keeping track when counting is essential in order to get an accurate count.

Math MileMarkers®

Draw a dot in each heart as you count. How many hearts are in this picture?



Teacher records the number of hearts counted. _____

Circle the number that tells **how many**.

1 2 3 4 5

Common Core Learning Standard

K.OA.3 Standard: Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

True Meanings

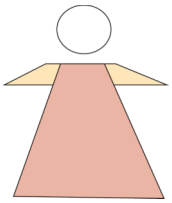
Grade Level: Kindergarten
Math Strand: Operations and Algebraic Thinking
Standard #3

True Meanings:

In standard K.OA.3, students work to develop their understanding that numbers can be broken into smaller subsets. For example, given 5 objects, students will learn that 5 can be broken apart (decomposed) into 2 parts and 3 parts, or 1 part and 4 parts, and then put back together to make the whole 5. Armed with this understanding, students will be able to find the unknown or missing part. Think about this scenario: Baley has 5 buttons on her dress; 3 are blue; how many are green? Understanding the *part-whole relationship* is an essential milemarker. Be careful not to move too quickly in introducing symbols. Most important at this MileMarker is that children use the word AND to generate the understanding that AND joins some objects with some more, a great connection to the future use of +, the addition symbol.

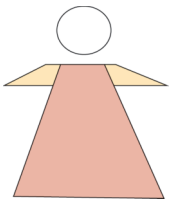
Math MileMarkers®

Marissa is missing 5 buttons on her dress. How many different ways can you use green and blue buttons to finish her dress?
Draw a picture of all your ideas.



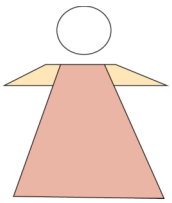
_____ blue ***and***

_____ green buttons



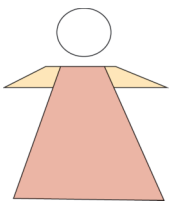
_____ blue ***and***

_____ green buttons



_____ blue ***and***

_____ green buttons



_____ blue ***and***

_____ green buttons

Common Core Learning Standard

Standard K.NBT.1: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8)*; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

True Meanings

Grade Level: K
Math Strand: Number and Base Ten
Standard #1

True Meanings:

It's really important to understand the progression of learning that happens when children develop understanding of the base ten system. In kindergarten, children use 10 objects to represent "10" rather than creating a unit called a ten, which is called unitizing. In first grade students learn to look at a bundle of ten ones as one ten. The writing of equations in kindergarten is encouraged but is not a required skill.

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Use objects or drawings to compose numbers 11 to 19. When organizing 16 objects, compose to 10 and add six more.

Build this number!

_____ tens and _____ ones

Build this Number!

_____ tens and _____ ones

Common Core Learning Standard

1.OA.1: Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

True Meanings

Grade Level: 1
Math Strand: Operations and Algebraic Thinking Standard #1

True Meanings:

By grade 1, students are expected to extend their experience of working with numbers to 20, and be able to use that knowledge to solve real-world problems. This is a great time to introduce or reinforce part/whole templates that will be the go-to structure for problem solving moving forward.

Many of the problems that fall within this standard ask students to **compare** how many more or how many less. In many cases, children are expected to find an unknown quantity.

People for years have mistakenly given too much focus to the words and not to what the numbers represent.

The US Department of Education recommends that in order to be successful problem solvers, students need to find and use common structures.

Let's take a close look at the sun and moon question to the right. Visually, students should be able to see that there are more suns than moons, but each child might approach the problem from one of the following angles:

1. The Counting On/Counting Up Method: Students count two moons and count up to 10 to identify how many more.
2. Make Five/Make Ten Method: Students recognize numbers in groups so they can easily make a five or ten. They visually see that they need 3 more moons to make 5 or they see the moons in terms of completing a ten frame, 3 more moons on the top line and 5 more below to make ten.
3. Structure Method: Students accurately assess information given and use structures such number bonds, part/whole structures, or bar and tape models to arrive at their answer.
4 TOTALLY Get It!: Students seek to find if the information provided gives them a total and then works with join templates or part/whole structures to arrive at their answer.

Students require extensive experiences with addition and subtraction situations in order to connect the experiences with symbols (+, −, =) and equations.

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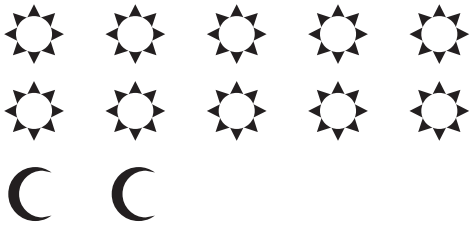
Problem Solving Situations—1.OA.1

1 How many more X do you need to make ten?

7 and _____ more make 10.

X	X	X	X	X
X	X			

2 Based on the picture below, ***how many more*** suns are there than moons ?



There are _____ more suns than moons.

3 Baley bought 11 chocolate bars and 7 lollipops at the store. How many chocolate bars and lollipops did Baley have all together?

Show your work:

Common Core Learning Standard

1.OA.3: Apply properties of operations as strategies to add and subtract.

- Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.)
- To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) . Students need not use formal terms for these properties.

True Meanings

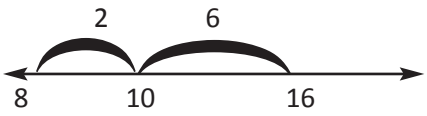
Grade Level: 1
Math Strand: Operations and Algebraic Thinking Standard #3

True Meanings:

Parents often talk about new math, but this standard provides proof that old math is alive and well and living within the common core standards. 1.OA.3 addresses important math strategies like:

- Commutative Property of Addition —the order of the addends does not change the sum.
- Associative Property of Addition— The grouping of the 3 or more addends does not affect the sum.

1 Use the number line to solve problems.



2 Use mental math strategies.
I know 5 can be decomposed (3+2=5)
I know 7 and 3 make 10
I know 2+2 more make 4
10 plus 4 more equals 14

It is essential that math students begin to utilize common structures and strategies that help them work through various problems. The number line and join templates that promote part/whole relationships can help students navigate through problem-solving situations including problems with unknown quantities.

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Problem Solving Situations—1.OA.3

1 Nicole had some marbles when she came to school today. She gave 5 marbles to Alyssa. Now she has 11 marbles. Write and solve an equation to show how many marbles Nicole had when she came to school today.

Show your work:

2 Maggie wants to solve the problem 8 + 6 + 2. She knows that if she can build a ten, it will be much easier to solve. Show how Maggie might solve the following equation. First circle the numbers that add to ten and then complete the equation.

(8) + 6 + (2) = 10 + _____ = _____

3 John wants to solve the problem 5 + 7 + 2. He knows that if he can build a ten, it will be much easier to solve. Show how John might solve the following equation by first breaking apart numbers to make a ten.

7 + 5 + 2 = 7 + ___ + ___ + ___ = 10 + ___ = ___

7 + 5 + 2
3 2

Common Core Learning Standard

1.NBT.2: Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
a 10 can be thought of as a bundle of ten ones—called a "ten."

True Meanings

Grade Level: 1
Math Strand: Number and Base Ten
Standard #2

True Meanings:

It is in first grade that students are formally introduced to the idea that a bundle of ten ones is called "a ten." This is called unitizing. Children who have mastered this skill see a group of ten ones as a whole unit ("a ten"), and they are able to count that group as if they were counting individual units. For example, 4 stacks of ten tiles has a value of 40. This concept is often very difficult for children but when mastered creates a monumental shift in understanding. This standard hinges on a student's ability to conserve numbers.

To further explain, take the task of separating 32 cubes. Students must learn to group these 32 objects in multiple ways:

3 groups of ten and 2 extra
2 groups of ten and 12 ones
1 group of ten and 22 ones

A student's ability to conserve numbers is an important aspect of this standard. It is not obvious to young children that 42 cubes is the same amount as 4 tens and 2 leftovers. It is also not obvious that 42 could also be composed of 2 groups of 10 and 22 leftovers. Therefore, first graders require ample time grouping proportional objects (e.g., cubes, beans, beads, ten-frames) to make groups of ten, rather than using pregrouped materials. It takes time and multiple experiences that extend past calendar time to master this understanding. Concrete manipulatives are a must!

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Performance Task 1.NBT.2

How many different ways can 32 cubes be grouped and still remain a total of 32 cubes? Use 32 cubes to build the number 32 and uncover the answer.

Common Core Learning Standard

Standard 2.OA.2: Add and subtract within 20.
Fluently add and subtract within 20 using mental strategies. By end of grade 2, **know from memory all sums of two one-digit numbers**.

True Meanings

Grade Level: 2
Math Strand: Operations and Algebraic Thinking Standard #2

True Meanings:

By grade 2, students need to internalize basic facts and use their knowledge of combinations of 10 to solve addition and subtraction facts to 20.

Mental Strategies that work

- Counting on
- Making ten
  (8 + 6 = 8 + 2 + 4 = 10 + 4 = 14)
- Decomposing a number leading to a ten
  (13 – 4 = (13 – 3) – 1 = 10 – 1 = 9)
- Using the inverse relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 − 8 = 4) Be familiar with fact families.
- Creating equivalent but easier or known sums (adding 6 + 7 by creating a known) 6 + 6 + 1 = 13

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A group of friends wanted to make a chalk mural at the park. Sammy brought 8 pieces of color chalk to the park to share with his friends. Ben brought 9 pieces of color chalk to the park to share with his friends. Only 11 pieces of chalk were used to create the mural. How many pieces of color chalk were not used?
Show your work using pictures, numbers, or words.

Mentally solve this problem:
Jimmy wants to buy a new shirt for $20. He earned $13. How much more money does Jimmy need to buy the shirt?

Explain using words how Jimmy can figure out how much more money he needs to buy the shirt.

Notes

1. D. Frye, A.J. Baroody, M. Burchinal, S.M. Carver, N.C. Jordan, & J. McDowell, (2013). *Teaching Math to Young Children: A Practice Guide* (NCEE 2014-4005). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, US Department of Education. Retrieved from the NCEE website: http://whatworks.ed.gov.

2. Common Core State Standards for Mathematics. Washington, D.C.: National Governors Association Center for Best Practices (NGA Center), 2010. Accessed at http://www.corestandards.org/Math/.

3. D. Frye, A.J. Baroody, M. Burchinal, S.M. Carver, N.C. Jordan, & J. McDowell, (2013). *Teaching Math to Young Children: A Practice Guide* (NCEE 2014-4005). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, US Department of Education. Retrieved from the NCEE website: http://whatworks.ed.gov.

We hope you have enjoyed the mathematical journey outlined in *Miss Penny Say's Prove It*. If you have any questions or comments or would like to learn more about other products and professional services, please contact us at mathmilemarkers.com.

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Additional Resources

1. Frye, D., Baroody, A.J., Burchinal, M., Carver, S.M., Jordan, N.C., & McDowell, J. (2013). Teaching Math to Young Children: A Practice Guide (NCEE 2014-4005). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, US Department of Education. Retrieved from the NCEE website: http://whatworks.ed.gov
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14. Rita Janes, Elizabeth Strong. Numbers and Stories: Using children's literature to teach young children number sense (April 2014).

The Author

Lynda Brennan has been in the educational arena for over twenty-three years. After many years as an elementary classroom teacher, she now serves as a math specialist in a K-5, title I, New York State public school. In addition to curriculum and assessment responsibilities; she works closely with teachers providing ongoing professional development, and with students working to build strong foundational skills and in-depth understanding of common core and state standards.

Lynda Brennan is an educational speaker and the creator of the Math Mile-Markers® game and book series. She has been featured at national teacher conferences and her message on how to *Calm, Command and Conquer the Curriculum*® has been well received. With degrees in both Marketing and Education, she has found a way to join her experiences and bring her love of learning to life for children and the adults who support them.