Charlie in Fraction City

EDUCATORS GUIDE

Lynda Brennan

A comprehensive educational resource aimed at demystifying Common Core and State Standards and promoting in-depth understanding of key math concepts for educators, parents and kids.

Math MileMarkers®
Calm, Command and Conquer the Curriculum®
Math MileMarkers Educational Guide/Children's Book Series
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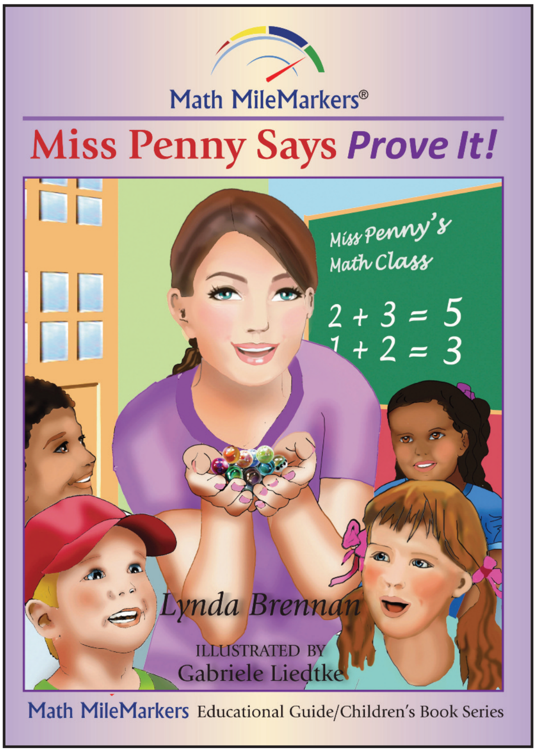
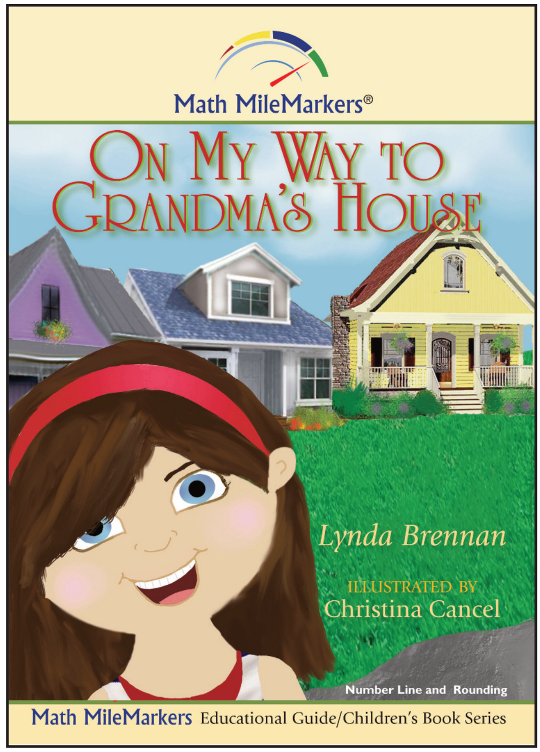
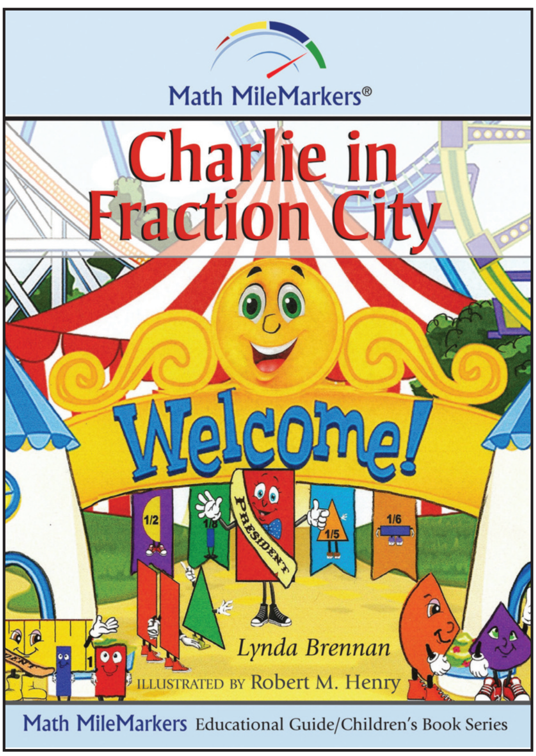
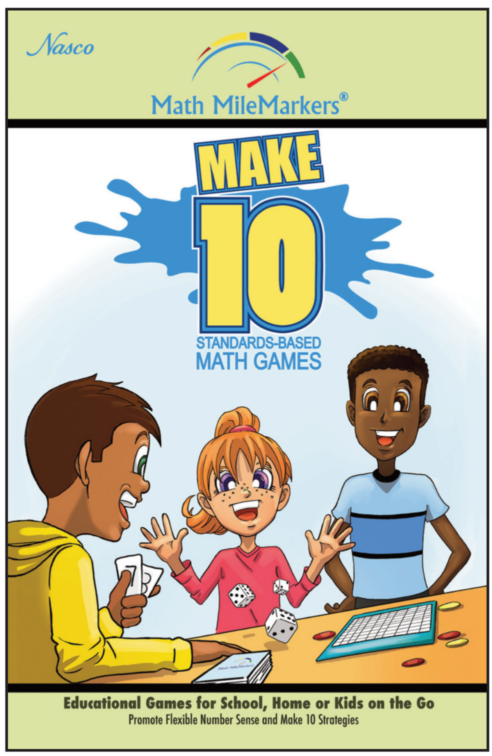








www.mathmilemarkers.com

Charlie in Fraction City
Educators Guide

Math MileMarkers® is proud to announce the latest addition to its math-infused children's books series entitled *Charlie in Fraction City*. The series was designed to help teachers and students Calm, Command and Conquer the Curriculum®. By integrating a simple story combined with a detailed educational guide, the Math MileMarkers series helps to build strong foundational skills and a deeper understanding of the math concepts that are embedded in Common Core and state standards.

The educational guide that follows our story outlines a standards-based continuum of learning. It addresses many of the missteps children often make on their path toward mastering the concept of fractions. As is true of all Math MileMarkers books, our main objective is to streamline current information related to math standards, allowing parents and teachers to get to the core of what children really need to know and understand in order to be successful in today's mathematical world. Our most important goal, however, is to make math meaningful and fun for all levels of learners.

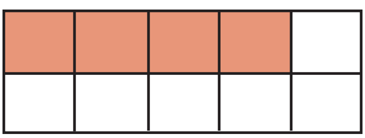
Who wouldn't love Charlie, a loveable rectangular character who happens to be the president of one of the designated *whole* clubs in Fraction City? You see, identifying with a whole is what gives fractional pieces their value, their true identity. All too often students miss this point and thus get confused when unit fractions are introduced and when students are asked to calculate or compare fractions. Charlie and his whole club president friends help provide a visual that supports this initial idea about fractions and fraction equivalencies.

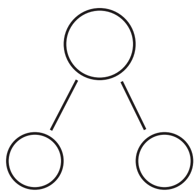
New resources such as *Cowboys Count, Monkeys Measure, and Princesses Problem Solve* and *Number Stories: Using Children's Literature to Teach Number Sense*, highlight the value of incorporating storytelling into math instruction as a means of fostering math inquiry and promoting the mathematical practice standards necessary for building true understanding. *Charlie in Fraction City* was designed to help launch math-rich discussions, encourage critical thinking and link content regarding fractions to standards in a meaningful way.

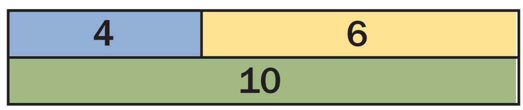
The information that follows outlines the journey that students take on the path to a solid k-5 fraction foundation. We are pleased to share our recommendations about some of the most respected resources available for teachers and students. Visit us at www.mathmilemarkers.com to share your ideas.

Number and Base Ten-Fractions: Where should one begin?

In teaching fractions, it's important to understand that the origin of this unit of study begins in the primary grades with lessons embedded in math strands such as Counting and Cardinality and Operations and Algebraic Thinking. In kindergarten and first grade, significant time is dedicated to building strong numeracy skills with emphasis on the composition and decomposition of whole numbers using concrete and visual supports like number bonds, bar/tape models, or part/whole structures. These are the early Math MileMarkers that outline the key instructional anchors along the way. Think for a minute of the ten frame and its connection to fractional understanding, 10 out of 10 representing a whole, or the visual created by a part/whole model as we teach students to find a missing piece. These visuals lay the foundation for fractions before the formal language, meaning the word "fraction," is ever uttered in the classroom.







In Grade 1, students are introduced to the concept of doubles and they begin to imagine a number divided into two *equal parts*, or a larger number built up from two equal parts forming a designated whole. This idea, supported by visual models like number bonds, softly ushers students into what will ultimately be fractional understanding that is based first on a solid understanding of whole numbers.

MileMarkers continue to emerge in Grade 2, when formal discussions surface about identifying equal parts of a finite space. Here students explore how geometric shapes like circles and rectangles can be divided into equal parts, with two halves, three thirds and four fourths all being equal to one whole. Again, concrete and representational models being the foothold for this understanding. Note that when the whole is presented on the bottom of part/whole model, the fractional pieces seem to jump out and announce themselves.

With this foundation in place, the concept of fractions and the related vocabulary becomes a natural transition in Grade 3. If done with these advancing steps as fractional understandings evolve, the formal introduction of fractions can be seamless and not so scary for students. Unit fractions and equivalencies become the obvious next step. More importantly, the application of this same key idea of designating a whole as a first step can be applied when children learn to identify fractional pieces on a number line.

By Grade 4, the understanding of fractional equivalencies is extended using area models and number lines as visual supports. From this work, students learn to compare fractional pieces, and use the same principles learned when adding and subtracting whole numbers to what in Grade 4 becomes the composition and decomposition of fractional pieces. Further piggybacking on prior skills, students are asked to multiply a whole number and a unit fraction, uncovering the reality that 4 x 3 = 3 + 3 + 3 + 3 just as 5 x ¼ = ¼ + ¼ + ¼ + ¼ + ¼. Repetitive addition and skip counting can still be a go-to strategy where this is concerned. Work with multiplicative comparisons using bar models or tape diagrams further supports the reasoning required to explore equal pieces as they relate to multiplication and division. Recognize again that there are clear Math MileMarkers that appear on each child's journey toward their own mathematical prowess. And because of the clear connection between these standards streaming from primary to intermediate grades, it's easy to differentiate instruction moving forward or back on the continuum as needed.

By Grade 5, the hope would be that students have developed number sense regarding fractions using benchmark understandings, such as how close a number is to 0 or 1, to guide their thinking and assess the reasonableness of their answers. With this in place they learn to add, subtract, multiply and divide fractions. And they begin to gain a better understanding of fractions as another representation of division. Once again students are charged with using their knowledge of whole number calculations to support this new concept and their fractional explorations.

This fractional evolution is eloquently scripted not just in today's common core and state standards, but in multiple resources whose roots were formed far before today's documents were written. These will be discussed further in the pages ahead.

Charlie in Fraction City was designed to grow with students, allowing younger students to connect with initial ideas about fractions and older students to explore more complex inquiries and related discussions. We encourage you to keep the mathematical practice standards alive as this topic unfolds. Join us as we explore Math MileMarkers' newest mathematical journey into the world of Fractions.

What Does the Research Say?

There are three highly respected resources that I believe all teachers, regardless of the primary teaching resource or text used, should reference as they create learning objectives and develop best practices for instruction related to fractions:

* The Rational Number Project- National Science Foundation
* The Progression Documents (Number and Operation/Fractions 3-5)– Arizona University
* Illuminations - NCTM

The Rational Number Project (RNP) is a research project funded by the National Science Foundation since 1979. This body of work promotes the use of models that encourage multiple representations of fractions with a comprehensive selection of lessons that support in-depth understanding of the initial ideas of fractions for children. In addition to individual lessons, the latest publication includes homework support, assessments, interviews and a collection of virtual manipulatives developed in conjunction with TPT (Twin Cities Public Television).[1]

The Progressions Documents, another go-to resource, was developed by the Institute of Mathematics and Science at the University of Arizona prior to the writing of common core and are tremendously insightful especially as it relates to the development of fractional understanding. This series of documents outlines how true learning of mathematics evolves from one topic to the next as well as the sequence in which that learning best meets the needs of students in the area of math. Seeing how the early phases of numeracy connect with future learning in mathematics is powerful for both teachers and students. According to this document, the basis for understanding fractions begins with recognizing a whole and being able to partition that whole into equal parts. It is this foundational understanding from which our main character Charlie was born.

According to the authors of the progression documents, "The goal is for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers; just as every whole number is obtained by combining a sufficient number of 1s, every fraction is obtained by combining a sufficient number of unit fractions."[2]

The components of the Mathematical Practice Standards are infused in both these resources, demanding not just that students be able to find an accurate answer, but also that they are able to justify that answer and expand their knowledge of fractions and all mathematical understandings for that matter by connecting prior learning and use of structures in a meaningful way. This idea of engaging students in activities and providing structures that promote in-depth understanding and encouraging each student's ability to communicate that understanding in a thoughtful way is further supported by the National Council of Teachers of Mathematics (NCTM).

Which leads to the third trusted resource, *Illuminations*, a program created by NCTM in conjunction with the Verizon Foundation's professional learning community known as Thinkfinity. *Illuminations* is a free on-line resources designed to support the teaching and learning of the K-12 math curriculum. This program includes interactive tools and games for students and instructional support materials for teachers. With games such as fraction feud and free apps for mobile devices, this NCTM/Thinkfinity project delivers on its promise to offer high quality standards- based materials to all.[3]

Of course there are a many more resources worth exploring including the Math Studio Talk videos presented by Engage NY,[4] which do an excellent job of explaining how visual models can be used to support fractional understanding, as well as the works of trusted math gurus such as Marilyn Burns, Kathy Richardson, Greg Tang, Jo Boaler, and so many more. A common thread is a strong belief in using visual models to enhance student learning.

1. http://www.cehd.umn.edu/ci/rationalnumberproject/default.html

2. See http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf, pages 3 – 14.

3. See https://illuminations.nctm.org/

4. See https://www.engageny.org/resource/math-studio-talk-common-core-instruction-4nf

Applying The Learning Standards

Common Core Learning Standards

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

True Meanings	Math MileMarkers®
Grade 1 **Math Strand:** **Operations and Algebraic Thinking** **Standard 6** In Grade 1, students continue to work with addition and subtraction, building numbers up and taking them apart. They begin to recognize that one whole can be combined with other whole numbers to create a collective sum of whole numbers. The addition of whole number units is an essential prerequisite skill for adding and subtracting fractional units in later years.	Eduardo had 4 chocolate chip cookies and 6 vanilla cream cookies. How many cookies did Eduardo have in all? There were 15 cookies; Parker ate 6 cookies. How many cookies were left?

Common Core Learning Standards

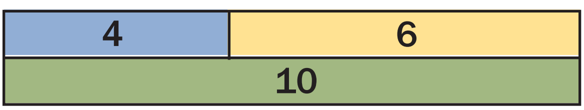
2.OA.A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

True Meanings

Grade 2
Math Strand: Operations and Algebraic Thinking
Standard 1

In Grade 2, students are afforded time in the curriculum to strengthen their understanding of whole numbers. They practice adding numbers up and taking them apart utilizing whole numbers to 100. The addition of whole number units is an essential prerequisite in the continuum and directly connects with the foundational understanding necessary to be able to add and subtract fractional units in later years. This also applies to identifying an unknown part of a given whole.

The concept of parts adding up to a whole is an essential understanding that should continue to be supported with visual models. This allows students to better understand the idea of parts adding up to a whole. When you look at it in this light, you can see how this simple part/whole structure supports fractional understanding.



In the part/whole structure above you can see that the blue section represents 4 units out of a total of 10 units. The yellow section represents a part that is 6 units out of a total of 10 units. Although not explicitly taught at this juncture, one can see that this lays the foundation for fractional understandings to come, making the visual model and its connections more powerful than what one might initially think.

$$\frac{4}{10} + \frac{6}{10} = \frac{10}{10}$$

Math MileMarkers®

There are 43 children at the school fair. 26 more show up. How many children are at the fair now?

Angelina has 56 photos that can be used for her school project. Her mom brings her some more photos. Now she has 82 photos. How many photos did her mom bring?

Shyla had some gummy bears in a bowl. She reached in the box and took out 23 more gummy bears to put in the bowl. Now Shyla has 36 gummy bears. How many gummy bears did Shyla have in the bowl at the start?

Common Core Learning Standards

2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

True Meanings

Grade 2

Math Strand:

Operations and Algebraic Thinking

Standard 4

The idea of equal addends provides yet another significant mathematical experience students must have on their way toward establishing true fractional understanding. Whole numbers are again used to solidify this idea conceptually, with models and structures to further support. Note that this standard, like standard 2.OA.3 which focuses on finding 2 equal addends in recognition of even and odd numbers, again emphasizes the important idea of finding "equal parts" that add up to a whole. Similarly, standard 2.G.2 asks students to divide a rectangle into rows and columns of same-size squares (equal parts) and count to find the total number of them.

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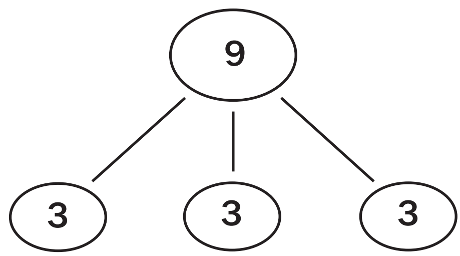
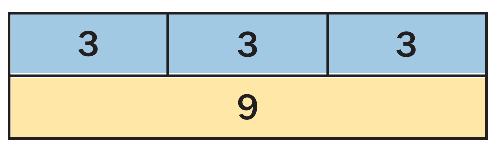
Write an equation that matches the total number of stars shown below?

Example: A student may say they see 3 rows with 3 stars in each row.

3 + 3 + 3 = 9





Common Core Learning Standards

2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

True Meanings

Grade 2

Math Strand: Geometry

Standard 3

Standard 2.G.3 marks the official entry into the world of fractions. The language and skill set supports fractional understanding by using geometric concepts, including classifying shapes by attributes, as a basis for their understanding.

One thought-provoking notion students must be exposed to at this juncture is that fractional parts may not always be symmetrical; in other words the pieces can look different. What makes each part equivalent is that the area is equal. Physical practice cutting and proving equivalency using pieces of paper is the best way to foster this important understanding. This standard builds directly on 2.G.2.

Vocabulary terms including halves, thirds, half of, third of, and fourth (or quarter), should be incorporated into all math talk and class discussion as students use paper and other manipulative tasks to explore and experience this concept.

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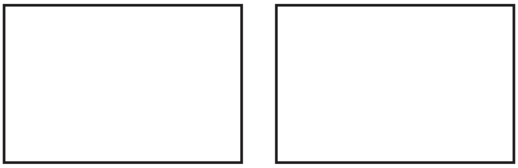
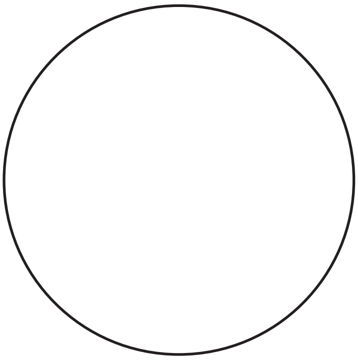
This rectangle represents one whole. You can divide this rectangle into 3 equal pieces. Show two ways to do that.



This circle represents one whole. Divide this circle into 4 equal pieces.



Common Core Learning Standards

3.NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

True Meanings

Grade 3

Math Strand:

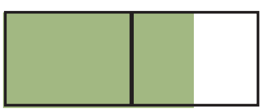
Number and Operations—Fractions

Standard 1

The story of Charlie was developed to help students comprehend the importance of visualizing a whole so that they can appreciate and recognize fractional pieces as part of that whole. Unit fractions (division into equal parts) are an essential piece of this understanding. Dividing a piece into equal pieces is often referred to as fair sharing. The essential understanding is that equal parts really mean parts of equal measure. Students use this newfound understanding of area models to reason about and support the idea that parts must be equal.

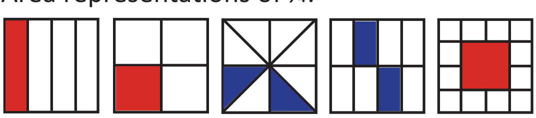
The Progression Documents outlines the following as a key understanding that must be addressed.

The importance of specifying the whole:



Without specifiying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction 3/2; if the entire rectagle is the whole, the shaded area represnets 3/4.

Area representations of ¼:



In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is 1/4 of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.

(Progressions for the CCSSM; Number and Operation – Fractions, CCSS Writing Team, August 2011, page 2)
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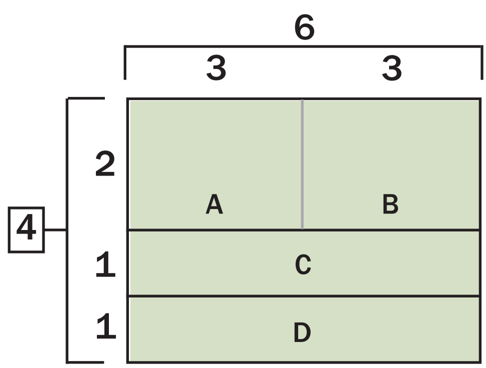
What is the whole? The shape below represents the unit fraction ¼. Draw a picture that would show what the whole might look like knowing the piece below is ¼ of the whole.

Daphne drew the array below. She told her friends that the section labeled A represented ¼ of the whole. Given this visual, is Daphne correct or incorrect?



Common Core Learning Standards

3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

True Meanings	Math MileMarkers®

Grade 3

Math Strand: Number and Operations-Fractions

Standard 2

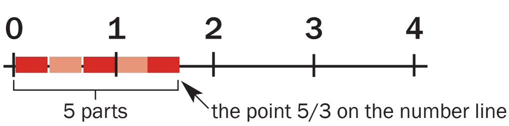
Up until this point, students have worked with number lines that focus on counting by whole numbers only. Standard 3.NF.A.2 ushers in the idea that there can be smaller pieces living between each whole number. We call these pieces fractions.

Number lines are often the most difficult aspect of fractions for students to grasp. However, if you return to the initial idea presented in the progression documents, which encourages as a first step that one must "identify the whole," the parts (fractions of the whole) become far easier to visualize.

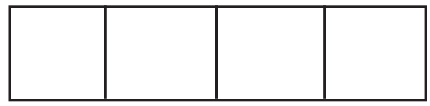
In the case of the ruler presented below, each whole number interval (0 to 1, 1 to 2, 2 to 3…) represents a finite space that is divided into three pieces, each of equal measurement. Using this model, students can see that the marking $1\frac{2}{3}$ is one whole plus two out of three pieces of the next whole. Armed with the understanding that each whole in this case is $\frac{3}{3}$ (three out of three pieces) students begin to make the connection as to why $\frac{5}{3}$ (an improper fraction) is equal to ($\frac{3}{3} + \frac{2}{3}$) and is therefore a number greater than one.

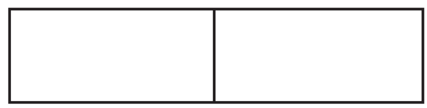
Number line representation of $\frac{5}{3}$:

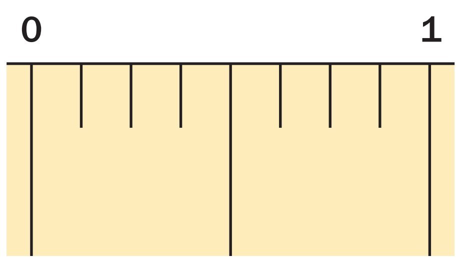
One part of a division of the unit interval into 3 parts of equal length.



Visualize a rectangle on top of the number line from O-1. Now students can divide that rectangle into equal parts similar to how they might divide a tray of brownies. This visual offers a far better view of fractional pieces on a number line. It is important to use this hands-on investigation of fractions to engage students in the mathematical practice standards, especially those that encourage them to look for and make use of structures, communicate their understanding, or provide justification for answers.







Common Core Learning Standards

3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

True Meanings

Grade 3

Math Strand: Number and Operations-Fractions

Standard 3

Fraction equivalency is a very important Math MileMarker for students. Allowing children time to visually explore this concept using concrete and pictorial models is a must. This standard is action packed as it incorporates multiple key understandings:

- Importance of identifying the whole
- Fractional Equivalency
- Exploring the True meaning of Denominator and Numerator
- Whole numbers written as fraction
- Fraction Comparisons

In order to compare fractions students' focus must again return to the first question, which is *what is the whole*? Comparisons are only valid if the wholes being compared are exactly the same. The example often given is ¾ of a small pizza cannot be *fairly* compared with ¾ of a large pizza because the wholes are different sizes. This makes for an important class discussion point as students reason about why two fractions that seem equivalent in writing are not equal at all.

In grade three, students are also asked to compare fractions with similar denominators with an understanding that the denominators is the number of equal pieces (pieces of same measurement) a given whole is divided into. One-fourth is therefore a smaller piece than ¾, the first being one unit out of 4 and the second being 3 units out of 4.

Additionally, they must come to a clear understanding that the whole number 3, can also be written as a fraction $^3/_1$ (3 wholes divided into one group.) This is an important foundational piece that will help students relate fractions to division as their journey with this topic continues.

Finally, this standard calls for students to compare fractions using visual models to support their understanding. Equally important are the opportunities students are given to talk about fractions.

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Pablo and his sister Malky each had 1 cookie. Pablo said that if he ate ½ of his cookie and Malky ate ½ of her cookie, that they would both have the same amount of cookie. Using the picture below explain why Pablo is correct or incorrect?

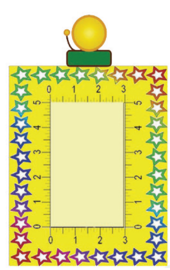
Pablo's Cookie

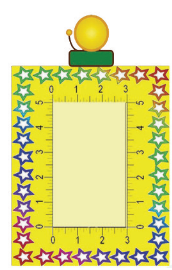
Malky's Cookie

Discuss a way that both children can share the two cookies equally. Is it possible for each child to eat the exact same amount?

Place ¼ and ⅓ on the number line below. Which fractional piece is bigger? Explain how you know?

Using Charlie's fraction meter as your whole, draw and name two equivalent fractions.





_______ = ______

Common Core Learning Standards

4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

True Meanings

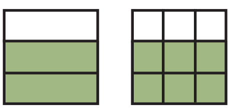
Grade 4

Math Strand: Number and Operations-Fractions

Standard 1

In Grade 4, the concept of fraction equivalents is expanded on as students are asked to create equivalent fractions by multiplying both the numerator and denominator by the same number or by dividing a shaded area into a different number of equal parts. Students begin to notice the power of fractions that are equal to one whole and the identity property

The starting point for all fraction inquiry hinges on the idea of “what is the whole?”



- What is the whole? (Whole)
- How many parts is the whole divided into? (Denominator)
- What region/part of the whole is shaded? (Numerator)
- What is the fraction of the whole? (Fraction)

This same language and sequence applies when children look at fractions on the number line. An opportunity to communicate and reason about fraction equivalency is essential.

Math MileMarkers®

Mrs. Kelly asked her class to bring in a box of crayons. By mid-year, some of the crayons had broken.

Ryan brought in a box of 10 crayons. There were 4 broken crayons inside his box. Rachel brought in a box of 20 crayons. There were 8 broken crayons inside her box.

What fraction of crayons was broken in Ryan’s box?

What fraction of crayons was broken in Rachel’s box?

Ryan noticed that the fraction of crayons that were broken inside his box was equivalent to the fraction of crayons that were broken inside Rachel’s box. How can this be possible if Ryan’s box has fewer crayons than Rachel’s?

Common Core Learning Standards

4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

True Meanings

Grade 4

Math Strand:

Number and Operations-Fractions

Standard 2

Using benchmark numbers like 0, ½ or 1 as a means of assessing the value of a fraction and identifying where a number lives in relation to other numbers on the number line, is very important. Let's take for example comparing $\frac{5}{8}$ and $\frac{6}{4}$. Students should be able to reason that $\frac{5}{8}$ is 5 pieces out of 8 and is therefore less than one whole. On the other hand, $\frac{6}{4}$ is an improper fraction. Because the numerator is greater than the denominator in this example, we know that it must be more than 1. To prove it, a student may explain that $\frac{6}{4}$ is that same as $\frac{4}{4} + \frac{2}{4}$, because $\frac{4}{4}$ is equal to 1 whole. The value of this fraction written as a mixed number is $1\frac{2}{4}$ or $1\ \frac{1}{2}$.

Math MileMarkers®

Three friends went on a bike ride through Central Park. They started at the zoo and traveled north on the path. They agreed to meet back as the zoo at noon.

Jackson traveled $\frac{6}{4}$ of a mile before he stopped to have a snack. Parker traveled $\frac{2}{8}$ of a mile before his snack break. Eduardo traveled $\frac{5}{8}$ of a mile before he stopped for his snack break.

Mark the distance each child traveled before their snack break. Record each fraction on the number line below.

Jackson claims that he traveled further than Eduardo and Parker combined. Show a comparison using symbols >, =, or <. Is Jackson correct? How do you know?

Common Core Learning Standards

4.NF.B.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

True Meanings

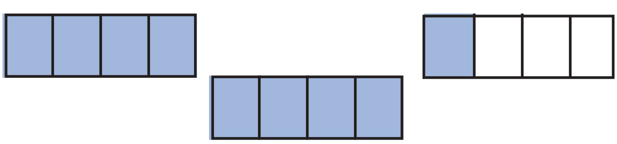
Grade 4

Math Strand: Number and Operations-Fractions

Standard 3

This standard is mainly about the composition and decomposition of fractions. It builds on the same foundational understanding that has been established in prior grades using whole numbers. In addition, it is a direct extension of standard 3.NF.3, where students learn to express numbers as fractions of a whole. Standard 4.NF.3 is the first time students are exposed to mixed numbers, a critical understanding that requires practice using concrete visual models. Understanding this concept once again hinges on a student's ability to identify the whole.

Given the number 2¼, students are expected to envision this mixed number as a combination of unit fractions. In order to do so, students must be able to identify the whole, in this case, 4 pieces out of 4 make up 1 whole.



Key understandings that must be established include the fact that one whole= $4/4$. This will allow children to further see that $4/4 + 4/4 + 1/4 = 9/4$, or ¼ + ¼ + ¼ + ¼ + ¼ + ¼ + ¼ + ¼ + ¼ = $9/4$, so 2¼ = $9/4$.

Math MileMarkers®

Chef Mario needed the following liquid ingredients for his strudel recipe:

- $3/4$ cup of melted butter
- $2/4$ cup of milk
- 1¼ cup of water

How many cups of liquid ingredients did he need altogether?

Draw a model to support your answer.

Explain how you can record your answer as both an improper fraction and a mixed number.

Common Core Learning Standards

4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

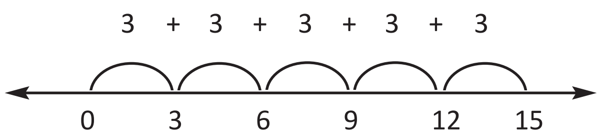
True Meanings

Grade 4

Math Strand: Number and Operations-Fractions

Standard 4

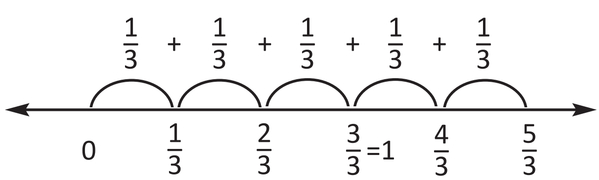
Once again, students will draw on the knowledge they gained about multiplication of whole numbers in grade 3 as their basis for understanding this standard. It is built on the premise that 5 x 3 can also be solved using jumps on a number line (skip counting) or with additive addition.



5 x 3 = 3+ 3 + 3 + 3 + 3
5 groups of 3 equals 15

When composing or decomposing fractions, students are expected to see how unit fractions can be used to build numbers up and take them apart.

$5 \times \frac{1}{3} = \frac{5}{3}$:



Math MileMarkers®

Six students in Mrs. Campbell's science class each added $\frac{1}{5}$ liter of water to a tub in preparation for a lab experiment they would be doing during their upcoming class.

Write and solve a multiplication equation and an addition equation that represent how the students might calculate the total amount of water in liters that the six students collectively added to the tub.

Addition equation

____

Multiplication equation

____

Common Core Learning Standards

5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of *fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

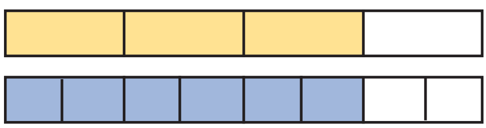
True Meanings

Grade 5

Math Strand: Number and Operations-Fractions

Standard 1

Standard 5.NF.1 builds off of NF standards presented in grade four which addressed finding equivalent fractions using bar models or other visual supports. This is an important start when first adding fractions with unlike denominators. It's important to overtly connect this to the work students did with standard 4.NF.1 where students came to the understanding that ¾ is equivalent to $6/8$ and further explored that connection by realizing that $3/4 \times 2/3 = 6/8$.



Using this foundational understanding, student can then explore addition and subtraction of fractions with unlike denominators.

$$\frac{3}{4} + \frac{4}{3} =$$

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$

$$\frac{4}{3} \times \frac{4}{4} = \frac{16}{12}$$

$$\frac{9}{12} + \frac{16}{12} = \frac{25}{12}$$

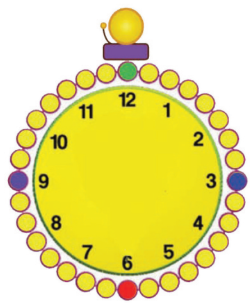
This is a great opportunity to relate the multiplication of fractions like $\frac{2}{3} \times \frac{2}{2}$ to the identity property which states any number time one does not change the value or identity of that number. That is why when we multiply $\frac{2}{3} \times \frac{2}{2}$ and get $\frac{4}{6}$, that $\frac{4}{6}$ is equivalent (measures the same area/has the same value as) to, $\frac{2}{3}$.

Note the answer from the sample above, $\frac{25}{12}$, can also be written as the mixed number $2\frac{1}{12}$ (see 4.NF.3). This is an important standard to review as students begin subtracting fractions in which they need to borrow or regroup from a whole.

When presented correctly, this standard prompts additional exposure to the mathematical practice standards.

Math MileMarkers®

Using the clock like model below, solve the following:



$$\frac{2}{6} + \frac{1}{3} =$$

$$3\frac{2}{4} + 2\frac{1}{6} + 1\frac{2}{3} =$$

$$6\frac{1}{3} - 2\frac{5}{12} =$$

Common Core Learning Standards

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product of (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3 × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)

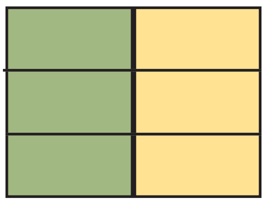
True Meanings

Grade 5

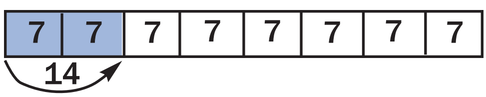
Math Strand: Number and Operations-Fractions

Standard 4

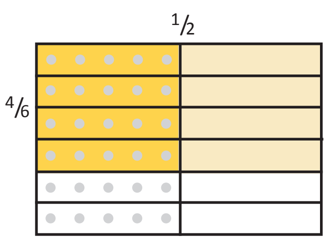
This standard covers a lot of ground with regard to the multiplication of fractions by fraction or whole numbers. Visual fraction models are the key to success for students.



In the case of the marching band problem presented on the right, students should see that the whole in this problem is represented by $^8/_8$. Again returning to the initial fraction understanding "what is the whole"? If $^2/_8$ of a number is 14, than each of the equal pieces that are part of that whole would be 7.



Similarly area models once used to show multiplication of numbers using the partial products strategy, can now be applied when multiplying fractions.



$^4/_6 \times {^1/_2} = {^4/_{12}}$

Math MileMarkers®

There were 14 new student members in the university marching band in 2016. These new performers represented $^2/_8$ of the entire band. What was the total number students that performed in the university marching band in 2016?

Common Core Learning Standards

CCSS.MATH.CONTENT.5.NF.B.6

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

True Meanings

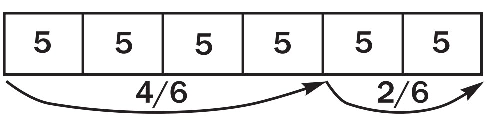
Grade 5

Math Strand: Number and Operations-Fractions

Standard 6

This standard builds on previous standards, and asks students to apply prior knowledge and skills to familiar strategies/structures for solving word problems.

In terms of approaching the Maggie problem presented, note the importance of using bar/tape diagrams in visually establishing the whole



Math MileMarkers®

Maggie bought $2\frac{1}{2}$ dozen cookies. Of these, $\frac{4}{6}$ were chocolate chunk and the rest were oatmeal raisin. How many oatmeal raisin cookies did she buy?

Mathematical Practice Standards:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Source: http://www.corestandards.org/Math/Practice/

Additional Resources

CCLS Standards: http://www.corestandards.org/Math

The Progression Documents: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf

Smarter Balanced Assessment, 4th Grade Mathematics: http://www.rcoe.us/educational-services/files/2013/11/asmt-sbac-math-gr4-sample-items.pdf

Engage NY: https://www.engageny.org/resource/math-studio-talk-common-core-instruction-4nf

Mathematical Practice Standards: http://www.corestandards.org/Math/Practice/

NCTM/Verizon: https://illuminations.nctm.org/

NC Unpacking Documents: http://www.dpi.state.nc.us/acre/standards/common-core-tools/

Math Studio Talk NYSED: https://www.engageny.org/content/math-studio-talk-common-core-instruction-video-series

Rational Number Project: (http://www.cehd.umn.edu/ci/rationalnumberproject/default.html

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About the Author

Lynda Brennan has been in the educational arena for over twenty-three years. After many years as an elementary classroom teacher, she now serves as a math specialist in a K-5, title I, New York State public school. In addition to curriculum and assessment responsibilities; she works closely with teachers providing ongoing professional development, and with students working to build strong foundational skills and in-depth understanding of common core and state standards.

Lynda Brennan is an educational speaker and the creator of the Math Mile-Markers® game and book series. She has been featured at national teacher conferences and her message on how to *Calm, Command and Conquer the Curriculum®* has been well received. With degrees in both Marketing and Education, she has found a way to join her experiences and bring her love of learning to life for children and the adults who support them.

About the Illustrator

Robert M. Henry is an accomplished American Artist and Illustrator. His career spans five decades. Now retired, Robert is living gracefully in West Virginia, where he only accepts commissions that greatly interest him. Thankfully, *Math MileMarkers: Charlie in Fraction City* was one of those. For Robert's partial on-line portfolio, please go to http://www.yourbasicwebpage.com/art. It is certainly worth a visit.

www.mathmilemarkers.com